

REACHABILITY TYPES

Tracking Aliasing and Separation in Higher-Order Functional Programs

SPLASH/OOPSLA 2021

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OWNERSHIP TYPE SYSTEMS

The “Shared XOR Mutable” Principle



OWNERSHIP TYPE SYSTEMS

Higher-Order Functions: “Counter” Examples

A Counter in { Scheme, ML, Scala,...} :

```
def counter(n: Int) = {  
  val c = new Ref(n)  
  (() => c += 1, () => c -= 1)  
}  
  
val (incr, decr) = counter(0)  
incr(); incr(); decr() // 1
```

Let’s Make One in Rust :

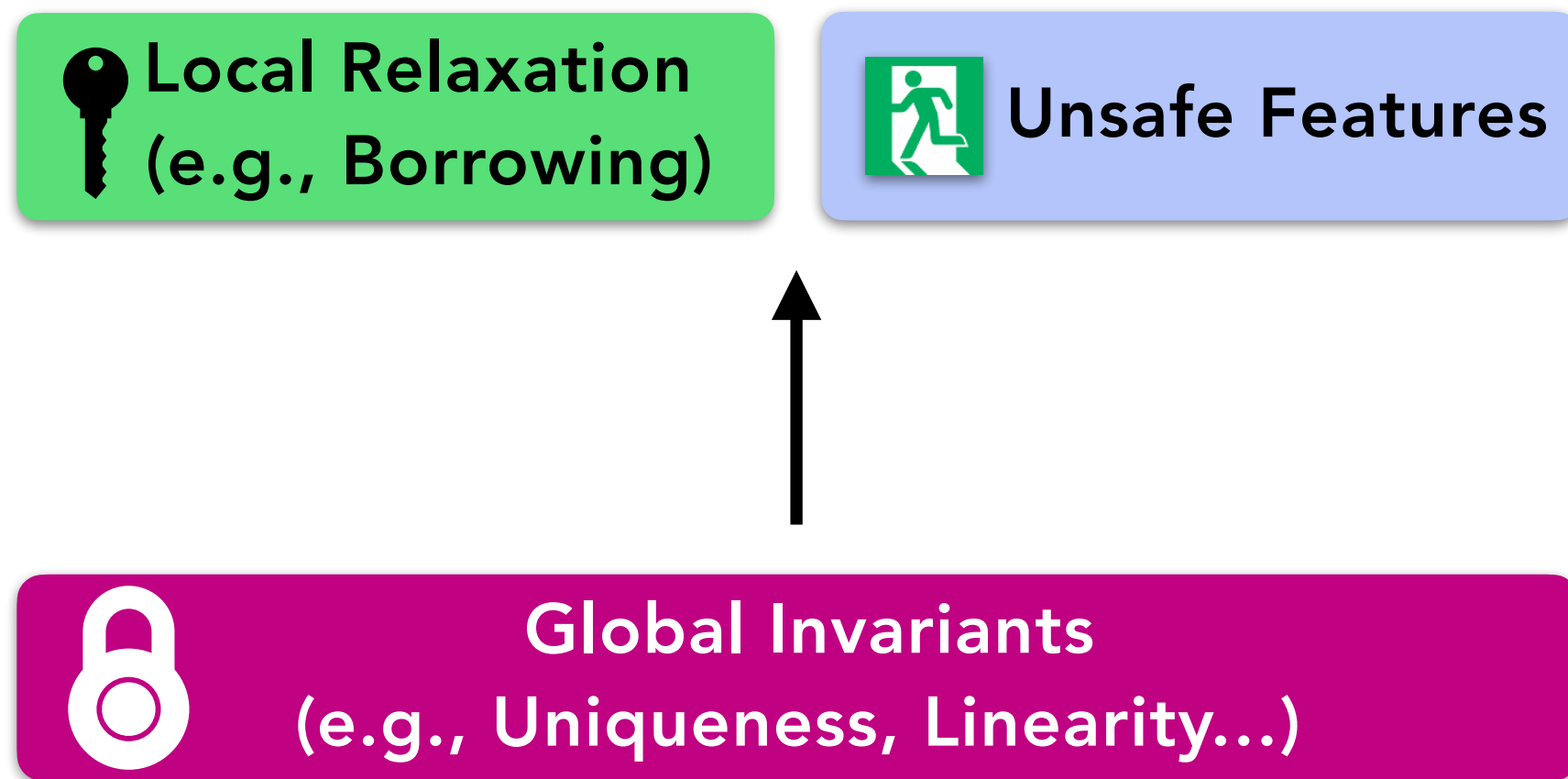
```
fn counter(n: i64) -> (impl Fn() -> (), impl Fn() -> ()) {  
  let c = Rc::new(Cell::new(n));  
  let c1 = c.clone();  
  let c2 = c.clone();  
  (move || { c1.set(c1.get() + 1); },  
   move || { c1.set(c2.get() - 1); })  
}
```

Dynamic reference counting,
no static lifetime tracking!

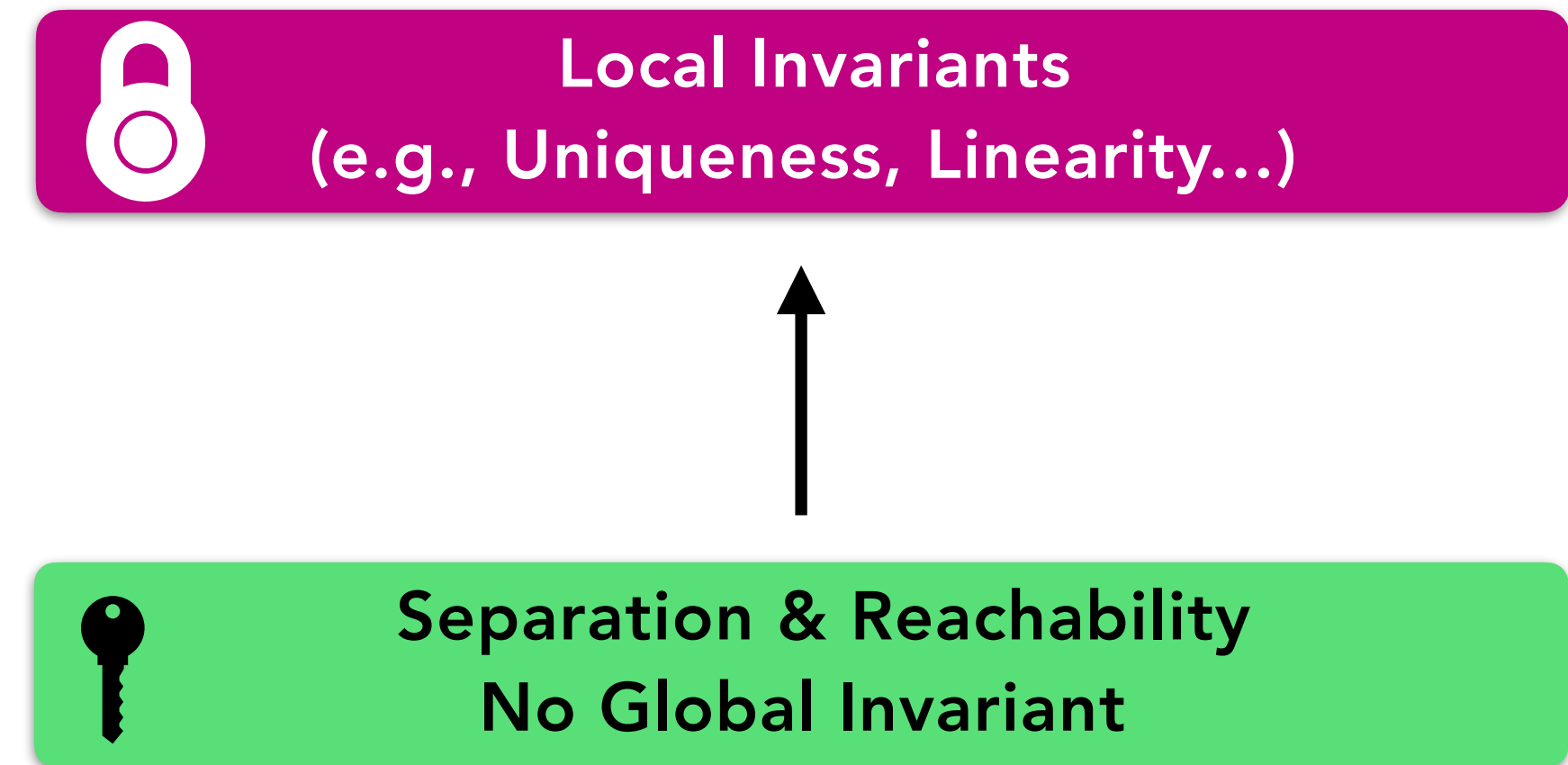


OWNERSHIP TYPE SYSTEMS

The Prevailing Ownership and Borrowing Model:



This Work Flips it on its Head:



RETHINKING OWNERSHIP IN TERMS OF SEPARATION LOGIC

RustBelt: Securing the Foundations of the Rust Programming Language

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- What happens if we expose separation and overlap all the way to the user-facing types?
- We get expressive ownership-style reasoning across higher-order functions!



REACHABILITY TYPES

THE λ^* CALCULUS

new Ref (42) : $\text{Ref}[\text{Int}]^\emptyset$

val $x = \text{new Ref}$ (42) : $\text{Ref}[\text{Int}]^{\{x\}}$

val $i = 42$: Int^\perp

val $y = x$: $\text{Ref}[\text{Int}]^{\{x,y\}}$

val $z = !y$: Int^\perp

$x := 0$: Unit^\perp

Intuition: Reachability Types & Qualifiers

$$\Gamma \vdash t : T^q$$

$$q \in \{ \perp \} \uplus \mathcal{P}_{\text{fin}}(\text{Var})$$

Computation t yields a T value which may reach all variables in q .

\perp is untracked (often omitted).

\emptyset means "fresh", no sharing w. context.

A simply-typed lambda calculus (STLC) with qualifiers, mutable references, recursion, and subtyping.

FUNCTIONS

Qualifiers Track Free Variables

```
val c1 : Ref[Int]{c1}; val c2 : Ref[Int]{c2}
```

addRef's implementation
reaches/closes over c1.

```
def addRef(c3 : Ref[Int]∅) = // (Ref[Int]∅ => Ref[Int]{c1}){c1}
```



```
addRef(c2) // ok
```

```
addRef(c1) // type error
```

addRef's implementation
must not share aliasing

with its argument: $\emptyset \sqcap \{c_1\} = \emptyset$

```
def addRef2(c3 : Ref[Int]{c1}) =  
  c1 := !c1 + !c3; c1
```

```
addRef2(c1) // ok now
```

```
// (Ref[Int]{c1} => Ref[Int]{c1}){c1}
```

Intuition: Observable Separation

- Functions track their free variables, consistent with view as closure records.
- To prevent interference from uncontrolled aliasing, functions are separated from their arguments
- If full separation is too strict, we may adjust the function domain's qualifier for degrees of overlap.

ESCAPING CLOSURES

How Can We Track their Free Variables?

Type Assignment **Inside** vs. **Outside** of Lexical Scopes

<code>{ () => new Ref(42) }</code>	<code>: (() => Ref[Int][∅])[∅]</code>	<code>~></code>	<code>{ () => Ref[Int][∅] }</code>
<code>{ val y = new Ref(42); () => !y }</code>	<code>: (() => Int){y}</code>	<code>~></code>	<code>{ () => Int }[∅]</code>
<code>{ val y = new Ref(42); () => y }</code>	<code>: (() => Ref[Int]{y}){y}</code>	<code>~></code>	<code>what now?</code>

Wrong: `{ () => Ref[Int]∅ }` returns a *fresh* reference on each call!

Right:

`f (() => Ref[Int]{y}){y}`
`<: f (() => Ref[Int]{f}){y}`
`~> f (() => Ref[Int]{f})∅`

Intuition: Function Self-Qualifiers

- Abstract over the free variables by letting a function type refer to itself. A concept borrowed from **DOT/Scala!**
- The self-qualifier's presence indicates that *some* qualifier escapes (existential statement).
- Subtyping (<:) makes their use ergonomic, compared to existential types.

LIGHTWEIGHT REACHABILITY POLYMORPHISM

def inc(x : Ref[Int][∅]) = { x := !x + 1; x } // : ((x : Ref[Int][∅]) => Ref[Int]^{x})[⊥]

Dependent function type! ↙ ↘

Lightweight Polymorphism (No Quantifiers!)

val c : Ref[Int]^{a,b,c} ; **val** d : Ref[Int]^{d}

inc(c) // : Ref[Int]^{a,b,c}

inc(d) // : Ref[Int]^{d}

inc(new Ref(0)) // : Ref[Int][∅]

Full details in the paper!

Reachability Types: Tracking Aliasing and Separation in Higher-Order Functional Programs 139:9

$t ::= c \mid x \mid \lambda f(x).t \mid t_1 t_2 \mid \text{ref } t \mid !t \mid t_1 := t_2$ $x, y, \dots, f, g, \dots \in \text{Var}$
 $T ::= B \mid \text{Ref } T \mid f(x : T^q) \rightarrow T^q$ $\alpha, \beta, \gamma \in \mathcal{P}_{\text{fin}}(\text{Var})$
 $\Gamma ::= \emptyset \mid \Gamma, x : T^q$ $q ::= \perp \mid \alpha$

Fig. 2. The syntax of λ^* .

$\frac{}{\Gamma \vdash c : B^\perp}$ T-CST	$\frac{}{\Gamma \vdash x : T^q}$ T-VAR	$\frac{}{\Gamma \vdash \text{ref } t : (\text{Ref } T)^\emptyset}$ T-REF	$\frac{\Gamma \vdash t_1 : (\text{Ref } T)^q}{\Gamma \vdash t_1 := t_2 : \text{Unit}^\perp}$ T-ASSIGN	$\frac{}{\Gamma \vdash !t : T^\perp}$ T-DEREF
$\frac{F = f(x : T_1^{q_1}) \rightarrow T_2^{q_2}}{(\Gamma, f : F^{q_f}, x : T_1^{q_1+x})_{q_f} \vdash t : T_2^{q_2}}$ T-ABS	$\frac{\Gamma \vdash t_1 : (f(x : T_1^{q_1}) \rightarrow T_2^{q_2})^{q_f}}{\Gamma \vdash t_1 t_2 : T_2^{q_2[q_1/x, q_f/f]}}$ T-APP	$\frac{\Gamma \vdash t_1 : T_1^{q_1} \quad \Gamma \vdash t_2 : T_2^{q_2}}{\Gamma \vdash t_1 <: T_2^{q_2}}$ T-SUB		
$\frac{\Gamma \vdash q_1 <: q_2 \quad \Gamma \vdash T_1^\perp <: T_2^\perp \quad \Gamma \vdash T_2^\perp <: T_1^\perp}{\Gamma \vdash (\text{Ref } T_1)^{q_1} <: (\text{Ref } T_2)^{q_2}}$ S-REF		$\frac{\Gamma \vdash q_1 <: q_2}{\Gamma \vdash B^{q_1} <: B^{q_2}}$ S-BASE		
$\frac{\Gamma \vdash q_5 <: q_6 \quad \Gamma \vdash T_3^{q_3} <: T_1^{q_1} \quad \Gamma, f : (f(x : T_1^{q_1}) \rightarrow T_2^{q_2})^{q_5+f}, x : T_3^{q_3+x} \vdash T_2^{q_2} <: T_4^{q_4}}{\Gamma \vdash (f(x : T_1^{q_1}) \rightarrow T_2^{q_2})^{q_5} <: (f(x : T_3^{q_3}) \rightarrow T_4^{q_4})^{q_6}}$ S-FUN				

Fig. 3. Typing and subtyping rules of λ^* .

$C ::= \square \mid C t \mid v C \mid \text{ref } C \mid !C \mid C := t \mid v := C$ $l \in \text{Loc}$
 $v ::= \lambda f(x).t \mid c \mid l \mid \text{unit}$ $\sigma ::= \emptyset \mid \sigma, l \mapsto v$
 $t ::= \dots \mid l$

$\frac{}{t \mid \sigma \rightarrow t' \mid \sigma'}$	$\frac{}{C[(\lambda f(x).t) v] \mid \sigma \rightarrow C[t[v/x, (\lambda f(x).t)/f]] \mid \sigma}$	$\frac{}{C[\text{ref } v] \mid \sigma \rightarrow C[l] \mid (\sigma, l \mapsto v)}$	$\frac{}{C[!l] \mid \sigma \rightarrow C[\sigma(l)] \mid \sigma}$	$\frac{}{C[l := v] \mid \sigma \rightarrow C[\text{unit}] \mid \sigma[l \mapsto v]}$	$\frac{}{C[\beta] \mid \sigma \rightarrow C[\beta] \mid \sigma}$	$\frac{}{C[l] \mid \sigma \rightarrow C[l] \mid \sigma}$	$\frac{}{C[l] \mid \sigma \rightarrow C[l] \mid \sigma}$	$\frac{}{C[l] \mid \sigma \rightarrow C[l] \mid \sigma}$
		$l \notin \text{dom}(\sigma)$ [REF]	$l \in \text{dom}(\sigma)$ [DEREF]	$l \in \text{dom}(\sigma)$ [ASSIGN]				

Fig. 4. Reduction Semantics of λ^* .

TYPE SOUNDNESS

Progress & Preservation [Wright & Felleisen '94]

Preservation

If $\emptyset \mid \Sigma \vdash \sigma$, $\emptyset \mid \Sigma \vdash t : T^q$, and $t \mid \sigma \longrightarrow t' \mid \sigma'$,

then $\emptyset \mid \Sigma' \vdash \sigma'$ and $\emptyset \mid \Sigma' \vdash t' : T^{q \oplus q'}$

for some $\Sigma' \supseteq \Sigma$ and $q' \sqsubseteq \text{dom}(\Sigma') \setminus \text{dom}(\Sigma)$

Corollary: Preservation of Separation

$\emptyset \mid \Sigma \vdash t_1 : S^{q_1} \parallel \emptyset \mid \Sigma \vdash t_2 : T^{q_2} \quad q_1 \sqcap q_2 \sqsubseteq \emptyset$



$\emptyset \mid \Sigma' \vdash t'_1 : S^{q'_1} \parallel \emptyset \mid \Sigma' \vdash t'_2 : T^{q'_2} \quad q'_1 \sqcap q'_2 \sqsubseteq \emptyset$

- Information may increase due to **fresh allocations**.
- Cancelling union ensures that untracked terms remain untracked: $\perp \oplus q = \perp$ $\alpha \oplus q = \alpha \sqcup q$
- Limitation: References must be *shallow*. We will solve this next.
- Interleaving two **computations** with **separate** answers keeps them separate.
- Reduction steps never introduce spurious aliasing/sharing between the two answers.

HIGHER-ORDER FUNCTIONS

Non-Escaping Values [Osvald et al. 2016]

```
def try[A⊖](block: (CanThrow⊖ => A⊖)⊖): Option[A]⊖
```



Return value cannot
close over the capability.

```
val c1 = new Ref(0)
```

```
try { throw =>
```

```
  c1 += 1
```

```
  if (error) throw(new Exception("legal"))
```

```
  () => throw(new Exception("illegal"))
```

```
}
```

- The base calculus supports effects as capabilities models and lightweight effect polymorphism [Brachthäuser et al. 2020].
- Reachability types alone do not capture linear consumption of capabilities, etc. This requires a proper effect system.

Non-Interference

```
def par(a: (() => Unit)⊖)(b: (() => Unit)⊖): Unit
```



Threads must have
non-overlapping qualifiers

```
val c1 = new Ref(0); val c2 = new Ref(0)
```

```
// ok, no overlap
```

```
par { c1 := !c1 + 1 } { c2 := !c2 + 2 }
```

```
// type error, overlapping
```

```
par { c1 := !c1 + !c2 } { c2 := !c1 + !c2 }
```

```
// type error, overlapping, but safe (!)
```

```
par { !c1 + !c2 } { !c1 + !c2 }
```

- Effect systems can help making more fine-grained distinctions.

II

REACHABILITY & EFFECTS

REACHABILITY-AND-EFFECT SYSTEMS

- Reachability sets permit very precise effect systems, at the granularity of variables, in both flow-insensitive and flow-sensitive flavors.
- Effects make reachability types powerful enough to enable ownership transfer, consumption policies (e.g., linearity), borrowing, etc.
- All we need are flow-sensitive “kill” effects to model nested references, consumption policies, move semantics, etc.

FLOW-SENSITIVE KILL EFFECTS

Enable Uniqueness, Linearity, Ownership Transfer & More

Example: Use-Once Functions from Self-Killing

```
// fun(Int =>({fun} : kill) String)⊘  
def fun(x) = { “Goodbye, cruel world!” }
```

```
fun(0) // fun at most once
```

```
fun(1) // type error, no more fun!
```

RECOVERING NESTED REFERENCES

Move Semantics and Ownership Transfer via Kill Effects

```
def f(x: Ref[Int]∅) = { val y = move(x); ... }
```

```
val z = new Ref(1)
```

```
f(z) // z is killed by f and unusable
```

```
!z // type error
```

CASE STUDIES IN THE PAPER

Reachability Types and Flow-(in)sensitive Effects for:

- Control Operators
- Algebraic Effects and Handlers
- Concurrency Combinators

$$\frac{\Gamma, k : (k(x : A^{q_1}) \rightarrow^{(\epsilon_k \triangleright KE)} B)^{\{k\}} \vdash t : A^{q_2} \mid \epsilon \quad NE}{\Gamma \vdash \mathcal{C} \ k \ \text{in} \ t : A^{q_1} \mid \epsilon}$$

Variants

let/cc: $B = \text{Nothing}^\perp$ shift: $B = C^{q_3}$

Attributes

Escaping (yes/no): $NE = \text{true}$ $NE = k \notin FV(A^{q_2})$

Affine (yes/no): $KE = \{(\{k\}, \text{kill})\}$ $KE = \{\}$

EFFECT QUANTALES

Polymorphic Iterable Sequential Effect Systems

COLIN S. GORDON, Drexel University

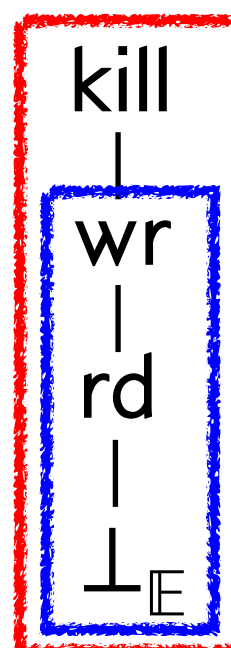
Effect Quantale [Gordon 2021]:

A structure $(\mathbb{E}, \sqcup, \triangleright, I)$ where
 (\mathbb{E}, \sqcup) is a *partial join semi lattice*, and
 $(\mathbb{E}, \triangleright, I)$ is a *partial monoid*.

Store-Sensitive Effect Quantale (New Here):

The lifting of a quantale $(\mathbb{E}, \sqcup, \triangleright, I)$
to a quantale over disjoint finite maps $\{\overline{(\alpha, \epsilon_{\mathbb{E}})}\}$,
assigning effects to reachability sets.

Example Effect Quantale:



Flow
sensitive

Flow
insensitive

\triangleright	$\perp_{\mathbb{E}}$	rd	wr	kill
$\perp_{\mathbb{E}}$	$\perp_{\mathbb{E}}$	rd	wr	kill
rd	rd	rd	wr	kill
wr	wr	wr	wr	kill
kill		undefined		

SUMMARY & CONTRIBUTIONS

Reachability Types

- Ownership-style reasoning for impure higher-order functional programs.
- Track sharing and its absence, inspired by separation logic.
- No global heap invariants.
- Statically safe, lightweight types & idiomatic code.

Reachability-and-Effect System

- Extensible effect system, based on store-sensitive effect quantales.
- All we need are flow-sensitive “kill” effects to model nested references, linearity, uniqueness, move semantics, etc.

Artifacts

- Interactive Prototype.
- Coq mechanization of variants of the base λ^* calculus.
- Available at:
<https://github.com/TiarkRompf/reachability>